Historical Contributions to Screw Theory

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Sir Robert Stawell Ball (1840-1913)

- A Treatise on the Theory of Screws (1900, 1998), (1876)
  - most comprehensive text on screw theory
- Chief proponent of screw geometry
- Screw theory was his avocation not his principal research

Sir Robert Stawell Ball (1840-1913)

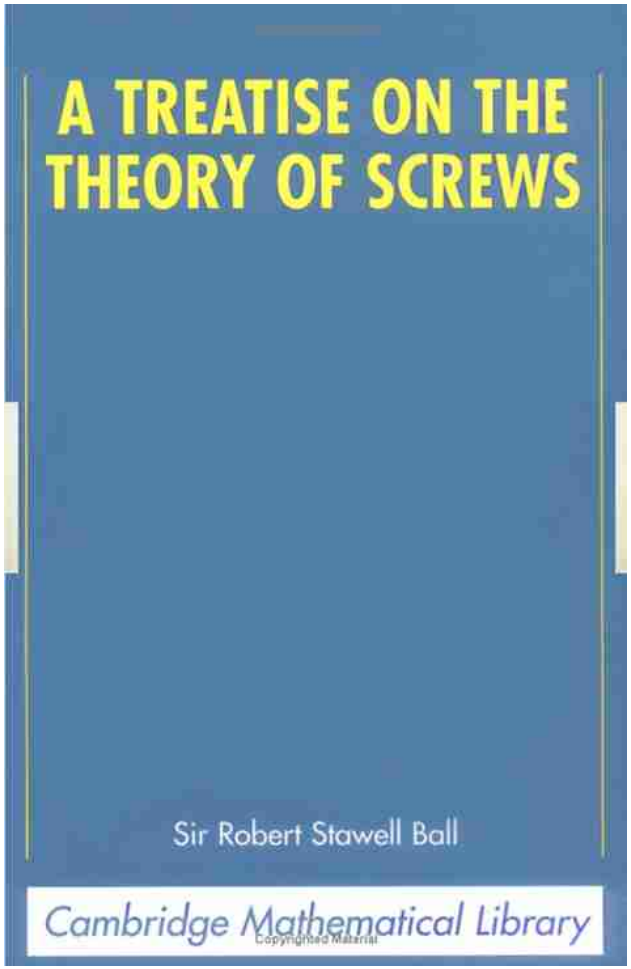
- Royal Astronomer of Ireland
  - Also held by W.R. Hamilton
- Lowndean of Astronomy and Geometry at Cambridge University
- Wrote many books on astronomy for the public
- Well-known public speaker

Sir Robert Stawell Ball (1840-1913)

- Screw kinematics and dynamics.
- Dynamics of rigid bodies at rest (linear geometry of dynamics)
- Uniquely employed *screw coordinates* – not currently used
- No known successor to further screw theory
  - Dynamicist E.T. Whittaker was a student

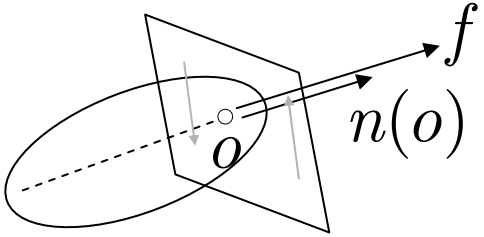
Sir Robert Stawell Ball (1840-1913)

- Died before finishing autobiography/biography – screw related materials not finished
- Ball 2000 Symposium at Cambridge University
- Treatise republished (1998)
- Extensive annotated bibliography "of principal works known to me"



Louis Poinsot (1777 – 1859)

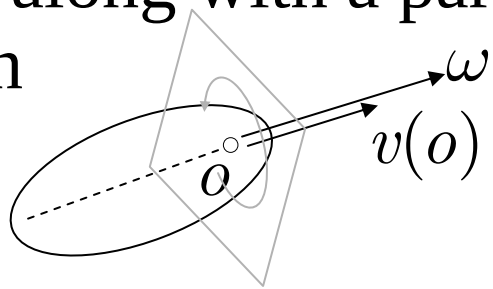
- Ecole Polytechnique
- Introduced geometric mechanics
- (1806)
  - concept of a **couple**
  - concept of a **wrench**: any system of forces can be reduced to a single force and a couple in a plane normal to the force



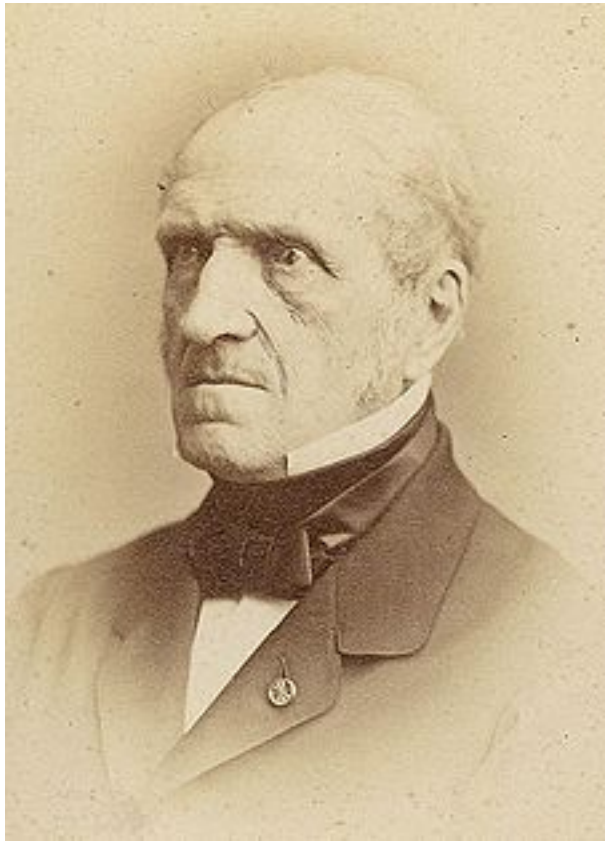
  - Ball: a wrench on a screw

Michel Chasles (1793 – 1880)

- Ecole Polytechnique
- Projective geometry
- (1830) concept of a **twist**
  - a rotation along with a parallel translation

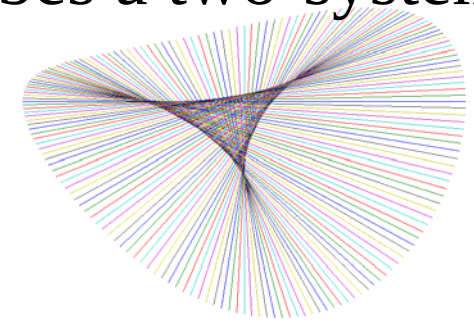


  - finite and infinitesimal twists
  - Ball: a twist on a screw



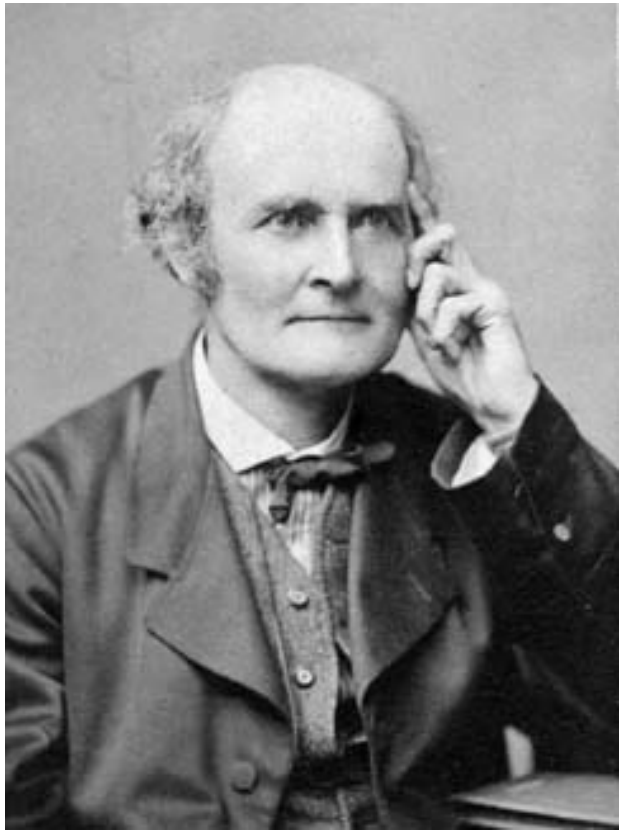
Sir William Rowan Hamilton (1805 – 1865)

- Royal Astronomer of Ireland (predates Ball)
- Famous dynamicist
  - Hamilton's principal
- Invented quaternions
- (1830) Discoverer of the cylindroid, the surface that describes a two-system of screws



Arthur Cayley (1821-1895)

- “High priest” of British mathematicians
- Geometry of *n* dimensions
- Linear algebra (w/Sylvester)
- (1859) Projective geometry most general
- (1860) Six coordinates of a line (predates Plücker)
- Lawyer for 14 years, then at Cambridge University



Hermann Grassmann (1809-1877)

- (1844) Theory of Extension
  - Basis of vector algebra and other branches of mathematics
  - General and abstract geometry
  - Little noticed
- (1862) Republished
  - used coordinates and determinant principles
  - Special cases relate to screw theory

Julius Plücker (1801-1868)

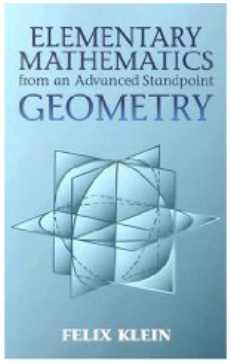
- (1831) Projective geometry
  - Developed principal of duality analytical
  - Use of homogeneous coordinates
- (1865-1869) Plücker coordinates
  - Ray and axis coordinates of a line using determinants
    - Special case of Grassmann's theory
  - Dynames (twists and wrenches)
  - Linear complex (five-system)

Felix Klein (1849-1925)

- Student of Plücker
  - (1868) finished Plücker's book on line geometry
- Defined geometry in terms of groups and invariants
- Completed Cayley's unification of projective and metrical geometries
- Screws as points in P^5
  - Lines lie on a quadric surface
- Klein coordinates (screws)
- (1878) edited Grassmann's collected works

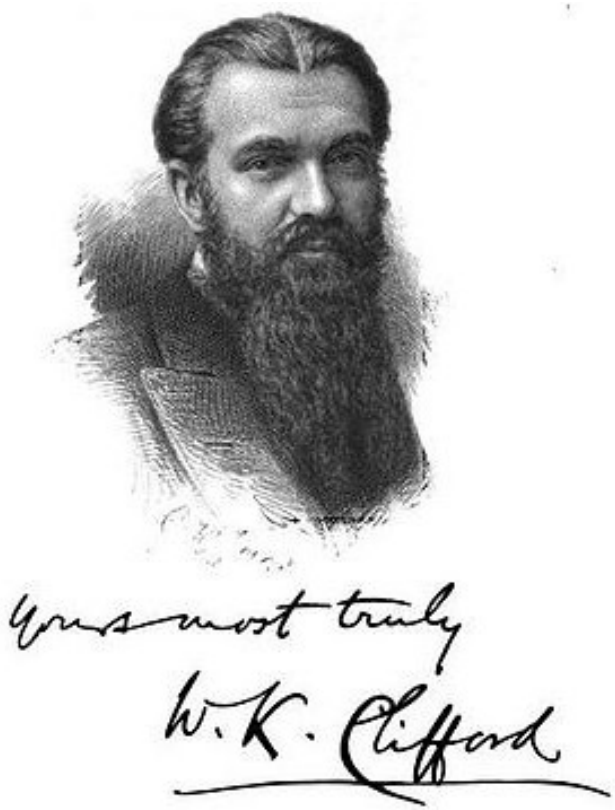
Felix Klein (1849-1925)

- Influential international leader in mathematics and mathematical education
  - (1908) Elementary Mathematics from an Advanced Standpoint: Geometry
    - Plücker coordinates, line complex, Grassmann principle, Erlangen program, Cayley's principle
    - Best text on 19th century geometry



William Kingdon Clifford (1845-1879)

- (1872) Biquaternions
  - Generalized quaternions to finite spatial displacements
  - Introduced “dual complex numbers”
    - $a + \omega b$ where $\omega^2 = 0$ (scalar)
    - $\boldsymbol{a} + \omega \boldsymbol{b}$ where $\omega^2 = 0$ (motor/dual vector)
  - Investigated elliptic geometry
- Clifford algebras
  - Generalization of Grassmann algebras
- Died age 33



Ball – Clifford (1870s)

- “In the early days of the ‘Theory of Screws’ Ball had made the acquaintance of W. K. Clifford at meetings of the British Association. They were drawn together by a common interest in geometrical forms, including non-Euclidean geometry; and both had a play of humour and frolic which made them for some years main upholders of the lighter side of the activities of that scientific body.”

  Sir Joseph Larmor from Ball’s autobiography

Ball – Clifford – Klein (1870s)

- “at a meeting of the British Association in the 'seventies, there turned up a young geometer from the University of Erlangen, Felix Klein, already a leader in the German mathematical world. Klein told them about Plücker's linear complex, and certain recent developments in the fascinating field of geometrical relations which it involved, a field in which Klein had first shown his own genius. Ball described how he and Clifford captured Klein after the meeting, and sat up half the night exchanging ideas, the interview culminating with the impatient and admiring complaint that there was positively nothing they could tell him that Klein did not seem to know about already.“

  Sir Joseph Larmor from Ball’s autobiography

Sophus Lie (1842-1899)

- (1872) Isomorphism between Plücker coordinates and sphere coordinates
- (1888) Lie groups and Lie algebras
  - Theory of continuous groups and tangent fields
  - Used to model finite and infinitesimal twists
  - Screw cross product is a Lie bracket
- College roommate of Klein

Aleksandr P. Kotelnikov (1865 – 1944)

- (1895) Developed screw theory using biquaternions
- Principle of transference between spherical and spatial geometry (using biquaternions / dual numbers)
- Influenced later Russian research

Eduard Study (1862 – 1930)

- (1903) Developed screw theory using biquaternions
- Principle of transference between spherical and spatial geometry (using biquaternions / dual numbers)
- Influenced later German research

Richard von Mises (1883 – 1953)

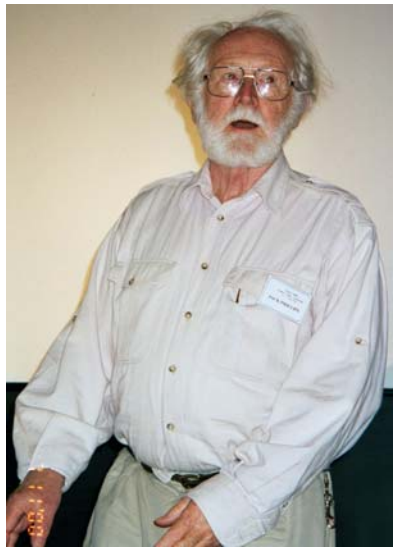
- Important contributions in the area of mechanics
  - E.g. von Mises stress, aerodynamics
- (1924) Motor Calculus
  - Very modern treatment with applications to mechanics
  - Influential on German research
- Married to Hilda Geiringer
  - (1942) Delivered lectures on Motor Calculus at Brown University

Louis Brand and Heinrich Guggenheimer

- Brand
  - (1947) Vector and Tensor Analysis
    - Has chapter on motor algebra
    - Develops motor/screw/spatial derivative
      - (with fundamental misprint latter corrected)
  - Advisor (Bôcher) was student of Klein
- Guggenheimer
  - (1963) Differential Geometry
    - Treatment includes the use of dual-numbers
    - Advisor of his advisor was a student of Klein

Kenneth H. Hunt (1920-2002) and Jack Phillips (1923-2009)

- (1962) “Rediscovered” the cylindroid and work of Ball
- Revived screw theory in 1960s
- Applied to spatial mechanisms
- Hunt
  - (1979) Kinematic Geometry of Mechanisms
  - (2004) Robots and Screw Theory (with J. Davidson)
- Phillips
  - (1984) Freedom in Machinery Vol.1
  - (1990) Freedom in Machinery Vol. 2
  - Based on synthetic (i.e. descriptive methods)



F. M. Dimentberg (1905?-1999?)

- Analysis of spatial mechanisms
- Used a hybrid of motor algebra and Plücker coordinates
- Screw properties of inertia and stiffness
- (1965) English translation of The Screw Calculus and Its Applications in Mechanics

Ferdinand Freudenstein (1926)-(2006)

- “Father of modern kinematics in the US”
- Analytical methods of mechanism design
- (1960s) Study of spatial mechanisms
  - Applied work of Dimentberg
- Revitalized screw theory in US along with many of his students including Yang and Roth
  - Popularized works of Brand, Klein, Ball, Guggenheimer, Dimentberg, and others